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Entire Functions Sharing a Second order Polynomial with its Derivatives

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Abstract: We prove a uniqueness theorem for an entire function, which share a function with their first and second order derivatives. We improve some existing results.
Keywords: Entire function, Polynomial, Uniqueness

1 Introduction, Definitions and Results

Let f be a non-constant meromorphic function in the open complex plane $\mathbb{C}$. We denote by $T(r,f)$ the Nevanlinna characteristic function of f and by $S(r,f)$ any quantity satisfying $S(r,f)=o\{T(r,f)\}$ as $r\to\infty$ except possibly a set of finite linear measure.

Let f and g be two non-constant meromorphic functions and let a be a complex number. We denote by $E(a;f)$ the set of a-points of f, where each point is counted according its multiplicity. We denote by $\overline{E}(a;f)$ the reduced form of $E(a;f)$. We say that f and g share a CM, provided that $E(a;f)=E(a;g)$, and we say that f and g share a IM, provided that $\overline{E}(a;f)=\overline{E}(a;g)$. In addition, we say that f and g share ∞ CM, if $\frac{1}{f}$ and $\frac{1}{g}$ share 0 CM, and we say that f and g share ∞ IM, if $\frac{1}{f}$ and $\frac{1}{g}$ share 0 IM.

For standard definitions and notations of the value distribution theory we refer the readers to [2]. However we require the following definitions.

Definition 1.1 A meromorphic function $a=a(z)$ is called a small function of f if $T(r,a)=S(r,f)$.

Definition 1.2 Let f and g be two non-constant meromorphic functions defined in $\mathbb{C}$. For $a,b\in\mathbb{C}\cup\{\infty\}$ we denote by $N(r,a;f\mid g\neq b)(\overline{N}(r,a;f\mid g\neq b)$ the counting function (reduced counting function) of those a-points of f which are not the b-points of g.

Definition 1.3 *Let f and g be two non-constant meromorphic functions defined in $\mathbb{C}$. For $a,b\in\mathbb{C}\cup\{\infty\}$ we denote by $N(r,a;f\mid g=b)(\overline{N}(r,a;f\mid g=b)$ the counting function (reduced counting function) of those a-points of f which are the b-points of g.*

In 1977 L.A.Rubel and C.C.Yang [7] first investigated the uniqueness of entire function sharing certain values with their derivatives. They proved the following result.

Theorem A *[7] Let f be a nonconstant entire function. If $E(a;f)=E(a;f^{(1)})$ and $E(b;f)=E(b;f^{(1)})$ for two distinct finite complex numbers a and b then $f\equiv f^{(1)}$.*

In 1979 E.Mues and N.Steinmetz [6] improved theorem A in the following manner.

Theorem B *[6] Let a and b be two distinct finite complex numbers and f be a nonconstant entire function. If $\overline{E}(a;f)=\overline{E}(a;f^{(1)})$ and $\overline{E}(b;f)=\overline{E}(b;f^{(1)})$, then $f\equiv f^{(1)}$.*